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DIFFRACTIONLIKE EFFECTS IN ANGULAR DISTRIBUTION OF CHERENKOV RADIATION FROM HEAVY IONS

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Formulae have been obtained for calculating the angular density of Cherenkov radiation from heavy ions with allowance for slowing-down of particle in the radiator. The diffraction structure of the angular spectrum of Cherenkov radiation (Fresnel diffraction) with the diffraction parameter dependent on the energy loss rates of particles is predicted.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Дифракционноподобные эффекты в угловом распределении черенковского излучения тяжелых ионов

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Получены формулы для расчета угловой плотности черенковского излучения тяжелых ионов с учетом торможения частиц в радиаторе. Предсказан эффект дифракционной структуры углового распределения черенковского излучения (дифракция Френеля) с параметром дифракции, зависящим от удельных потерь энергии.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Tamm — Frank's theory [1] describes Cherenkov radiation of a particle that moves steadily and rectilinearly in a medium with the refractive index $n > c/v$ (v is the particle velocity, c is the speed of light in vacuum). So, this theory ignores interaction between the particle and the radiator material, while this interaction results in particle trajectory bending (multiple scattering) and in systematically decreasing velocity (ionization losses).

Dedrick [2] was the first to take into account the influence of the particle-radiator interaction on the angular density of the particle's Cherenkov radiation. He examined the influence of multiple scattering, while ignoring the slowing-down effect.

Below we consider another limiting case where the multiple scattering effect is insignificant, and it is the slowing-down effect that plays a leading part in forming the angular distribution of Cherenkov radiation. It occurs when nuclei of heavy elements (Z, $A >> 1$) transverse thin radiators.

Ignoring fluctuations of energy losses, we shall assume that at each moment of time the particle has a very definite energy and thus a very definite velocity. Since we ignore the trajectory bending effect as well, our task is reduced to examining the angular distribution of the Cherenkov radiation emitted by a particle moving rectilinearly at a varying velocity. The answer can be found from the formulae of classical electrodynamics

$$\frac{dW}{d\,\omega\, d(\cos\theta)} = \frac{(ze\sin\theta)^2}{2\pi c^2}\, k\,\omega \left\{ \int_0^L \exp[i\,\Phi(x)\,]dx \right\}^2, \tag{1}$$

$$\Phi(x) = kx\cos\theta - \omega t(x); \quad t(x) = \int_0^x \frac{dx'}{v(x')}.$$

Here $dW/d\,\omega d(\cos\theta)$ is the spectral angular distribution of the radiation; $k,\ \omega$ are the wave number ($k = \omega\, n/c$) and the frequency of the Cherenkov photon; L is the thickness of the radiator; n is its refractive index, θ is emitting angle; ze is the ion charge.

Taking for thin layers of the radiator

$$\frac{1}{v(x)} \approx \frac{1}{v_0} - \frac{1}{v_0^2}\, v_0' x; \quad v_0' = \left.\frac{dv(x)}{dx}\right|_{x=0} < 0,$$

$$\frac{dv}{dx} = \frac{dE}{dx}\frac{dv}{dE} = \frac{1}{P}\left(1 - \frac{v^2}{c^2}\right)\frac{dE}{dx}; \quad P = Ev,$$

where $v_0 = v(0)$ is the particle velocity on entering the radiator; P, E are the momentum and the energy of the particle, we finally get

$$\frac{dW}{d\,\omega d(\cos\theta)} = \omega\, L\left(\frac{ze\sin\theta}{c}\right)^2 \cdot f(\theta,\omega),$$

$$f(\theta,\omega) = \frac{1}{2\Delta\,\theta\sin\theta_0} \cdot \{[C(u_1) - C(u_0)]^2 + [S(u_1) - S(u_0)]^2\}, \tag{2}$$

where

$$C(u) = \sqrt{\frac{2}{\pi}} \int_0^u \cos t^2 dt, \quad S(u) = \sqrt{\frac{2}{\pi}} \int_0^u \sin t^2 dt \;—$$

are Fresnel integrals, and

$$u_{0,1} = \frac{n\cos\theta - \omega/v_{0,1}}{a}; \quad a = \sqrt{-\frac{2\omega v_c'}{v_0^2}},$$

$$\theta_{0,1} = \arccos\frac{c}{nv_{0,1}}; \quad \frac{1}{v_1} = \frac{1}{v_0} - \frac{1}{v_0^2}v_1'L; \quad \Delta\theta = \theta_0 - \theta_1 \approx \frac{|v_0'|L}{v_0}\cot\theta_0,$$

$$\int_{-1}^{1} f(\theta, \omega)d\cos\theta = 1,$$

Here v_1 is the particle velocity on leaving the radiator. The radiator thickness being small, Cherenkov radiation is concentrated in a narrow angular interval $\theta_1 \leq \theta \leq \theta_0$, $(\theta_0 - \theta_1 \ll 1)$. Within this interval and in the close proximity to it variables $u_{0,1}$ can be expressed as

$$u_{0,1} = \frac{\tan\theta_{0,1}(\theta - \theta_{0,1})}{\theta_d}; \quad \theta_d^2 = \frac{|v_0'|\lambda}{\pi c},$$

where λ is the radiation wave length.

As an example, we give a distribution for the angular density of Cherenkov radiation as it follows from the Tamm — Frank theory [1]

$$f_{T-F}(\theta, \omega) = \frac{1}{\delta\theta}\left(\frac{\sin x}{x}\right)^2; \quad x = \frac{\pi}{\delta\theta}\left(\cos\theta - \frac{c}{v_0 n}\right). \tag{3}$$

The Tamm — Frank distribution width is defined as $\delta\theta = \lambda/nL$, where $\lambda = 2\pi c/\omega$ — is the radiation wave length.

In our opinion, distributions (2) and (3) essentially differ in the following features (see figs.1a, 1b).

As compared with distribution (3), the centre of gravity in distribution (2) is shifted toward smaller angles by $\Delta\theta/2$. The width of (2) is much larger than that of the Tamm — Frank distribution, and it increases proportionally with the radiator thickness. The opposite situation is typical of distribution (3), whose width decreases in inverse proportion to the ratiator thickness.

Speaking in terms of optics, we may compare the angular density of Cherenkov radiation from a steadily decelerating particle to the Fresnel slot diffraction, while radiation from a steadily moving particle is distributed in accordance with the Fraunhofer diffraction pattern. Moreover, if in the

Tamm — Frank theory the distance between maxima (the diffraction parameter) is determined by the wave length and the radiator thickness, $\delta\,\theta = \lambda / nL$ in distribution (2), the expression for the diffraction parameter involves energy loss rates in the radiator material

$$\theta_d^2 = \frac{|v_0'|\lambda}{\pi c} = \frac{1}{\pi \beta_0}(1 - \beta_0^2)\left|\frac{dE}{dx}\right|\frac{\lambda}{E_0}, \quad \beta_0 = \frac{v_0}{c}.$$

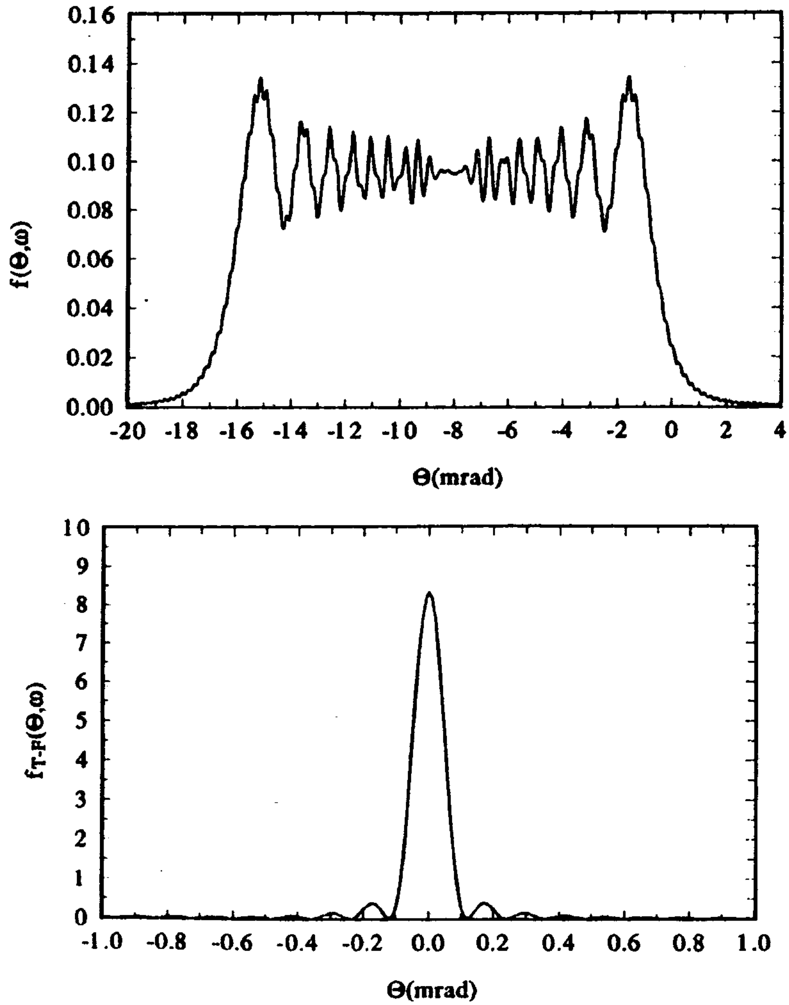

Fig.1. Distributions (2) and (3) calculated for gold nuclei of energy 1000 MeV per nucleon incident on a quartz radiator 0.5 cm thick. a) this paper; b) Tamm — Frank distribution

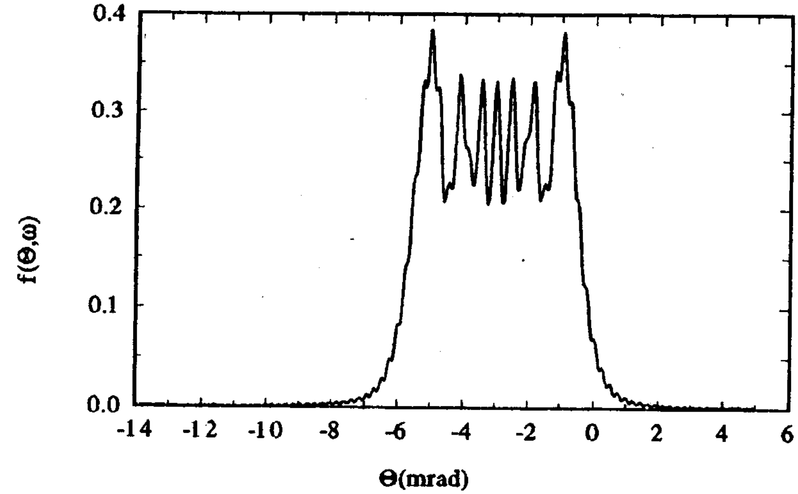

Fig.2. Distributions (2) calculated for iron nuclei of energy 1000 MeV per nucleon incident on a quartz radiator 0.5 cm thick

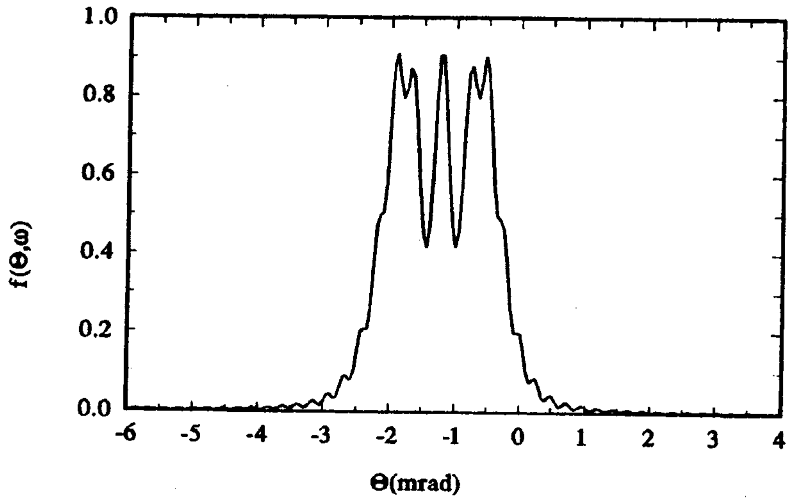

Fig.3. The same as in fig.2 but for neon nuclei of energy 1000 MeV per nucleon

The most distinct Fresnel-like diffraction pattern shows up in Cherenkov radiation of very heavy ions (uranium, gold), though its manifestations are observed with lighter nuclei as well.

In figs.2,3 there are angular distributions of Cherenkov photons with a wave length of $\lambda = 546.1$ nm emitted by iron and neon ions of energy 1000 MeV/nucleon. The photons are generated in a quartz plate of thickness $L = 5$ mm. Here a decrease in specific energy losses results in narrower angular distributions of radiation, but spectra keep their diffraction structure up to $Z = 10$.

The multiple scattering effects that we neglect may cause a partially or completely smoothed diffraction pattern predicted by formula (2). However, a more detailed analysis within the approach developed in ref. [3] shows that this negligence is justified to some extent when Cherenkov radiation from heavy ions of not very high energy (a few times hundred MeV per nucleon) is considered. The results of more complete investigations involving influence of both elastic and inelastic processes in the radiator will be published soon.

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